Bilateral Trading on a Network: A Simulation Study

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Abstract

This paper examines the dynamic behavior of a process of bilateral trading between spatially diverse agents. In contrast to the standard general equilibrium trading mechanism and to many previous models of bilateral trading, trades may only occur between agents who are located adjacent to one another. A network of bilateral trades provides one process by which an economy wide equilibrium can be achieved in a truly decentralized fashion. Computer simulations are used to explore the dynamic behavior of trading on a network. In particular, the speed of convergence to an economy equilibrium is examined for a variety of spatial structures. The process of convergence generates interesting intertemporal and cross-sectional dynamics such as the persistence of spatial correlations, or neighborhood effects, in the prices of the goods over time. In addition, the system exhibits path dependence: the equilibrium allocation and speed of convergence can depend crucially on details of the trading process such as the order of trades or location of agents on the network.

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1 Introduction

This paper explores the dynamic behavior and equilibrium characteristics of a regime of decentralized bilateral trade between spatially located agents. The standard competitive general equilibrium framework articulated by Arrow and Debreu [?], [?], [?] solves for market clearing prices for all goods at all times simultaneously. The model is decentralized from the point of view of individual decision making: all of the information an agent requires to solve his or her individual optimization problem is contained in an exogenously determined price. It also supposes a *tatônnement* process where no trades occur before the market clearing prices have been established. Hence, the competitive general equilibrium framework separates the establishment of economy-wide market clearing prices from the actual execution of exchanges between agents. Consequently, the equilibrium allocation does not depend on the particular features of the trading process such as the order in which exchanges occur. In contrast, this paper demonstrates that the details of the trading process matter when prices and individual exchanges are executed in a decentralized environment where trades may occur before market clearing prices have been determined.

The spatial structure of the model constrains agents' trading opportunities: trades may only occur between agents who are adjacent neighbors on a trading network. Trades occur at dispersed prices before economy wide market clearing prices are established, in other words, the model utilizes a non-*tatônnement* trading process. With appropriate restrictions on network structure and agent characteristics these mechanisms exhaust all possible gains from trade and converge to a globally Pareto efficient equilibrium distribution of goods (Bell [?]).

While convergence of the trading network is guaranteed, the decentralized sequential trading environment creates a wide range of dynamic behaviors. Some network structures exhibit smooth and fast convergence to an efficient allocation, while others generate interesting intertemporal and cross-sectional dynamics and exhibit extremely slow convergence. The average distance between all possible pairs of agents, a measure of network centralization, is a robust predictor of the speed of convergence. More centralized networks converge rapidly, in contrast, less centralized networks tend to converge slowly and often display persistent regional variations in prices over time.

Furthermore, the characteristics of the equilibrium allocation depend on the specific features of the trading process such as the network structure, agent characteristics and order of trades. A key assumption in the analysis is that agents are myopic and do not anticipate future trading opportunities; they only engage in trades that increase the utility of their current holdings of two infinitely durable consumption goods. Each trade narrows the set of potential equilibrium allocations, eliminating all allocations which would give either of the agents lower utility than the bundle they hold after trading bilaterally. The change in agents' holdings also shifts their future demand functions, affecting the future trajectory of the system. Because trades occur at dispersed prices each trade also changes the value of agents' holdings relative to the network-wide price that will ultimately prevail. Consequently the convergence process exhibits path dependence, and the equilibrium allocation can be crucially influenced by details of the trading process such as the order of trades. Another implication of the assumption of myopic agents is that the equilibrium allocation need not be in the core of the initial exchange economy—myopic agents may make utility improving trades that would have been unacceptable had they been fully informed about future prices and trading opportunities. Although every trade increases both agents' utility gains from trade can be quite unevenly distributed and often accrue disproportionately to agents in centralized locations on the network.

The analysis of networks in an economic context is receiving increasing attention. For example, a number of recent papers have examined the process of network formation: Jackson–Wolinsky [?], Kirman [?], Kirman–Oddou–Weber [?] and Bala–Goyal [?]. This paper takes the network structure as given, focusing on the behavior and outcomes associated with different networks. Understanding the properties of trading networks may help to explain the existence and importance of particular network structures, complementing existing work on strategic network formation. The dynamics of trading networks are analyzed through the use of computer simulations. Related simulation based models which analyze decentralized trading with boundedly rational agents include Albin–Foley [?] and Tesfatsion [?]. The former considers the interaction between search and communication in decentralized trading environment while the latter analyzes the endogenous formation of relationships between pairs of agents playing a prisoner's dilemma.

2 Definition of Goods, Agents and Networks

There are two infinitely durable consumption goods, (x, y). The goods are both a source of utility this period and next period's endowment or wealth. For example, the goods may be works of art or jewelry, valuable both as a source of enjoyment and as a durable asset.

Agents are myopic and maximize utility period by period subject to a budget constraint that reflects their current holdings of goods. This extreme assumption is based on prior theoretical treatments of the convergence of decentralized trading mechanisms. Furthermore, the assumption of myopic agents provides a useful benchmark for comparison with models with learning or with boundedly rational forward looking agents.

The number of agents is denoted n. For simplicity in the initial simulations all agents' preferences are represented by Cobb-Douglas unit income elasticity utility functions. The parameter α that determines relative preferences for the two goods varies across agents. This parameter is initially drawn from a uniform $[0.05, 0.95]$ distribution but is fixed for the entire simulation. Average preferences equal 0.5. Agent i maximizes utility $u^i(x_t^i, y_t^i)$ subject to a budget constraint:

$$\begin{array}{c} \max \quad x_t^{\alpha^i} y_t^{1-\alpha^i} \\ s.t. \; x_t^i + p_t y_t^i = x_{t-1}^i + p_t y_{t-1}^i \end{array} \tag{1}$$

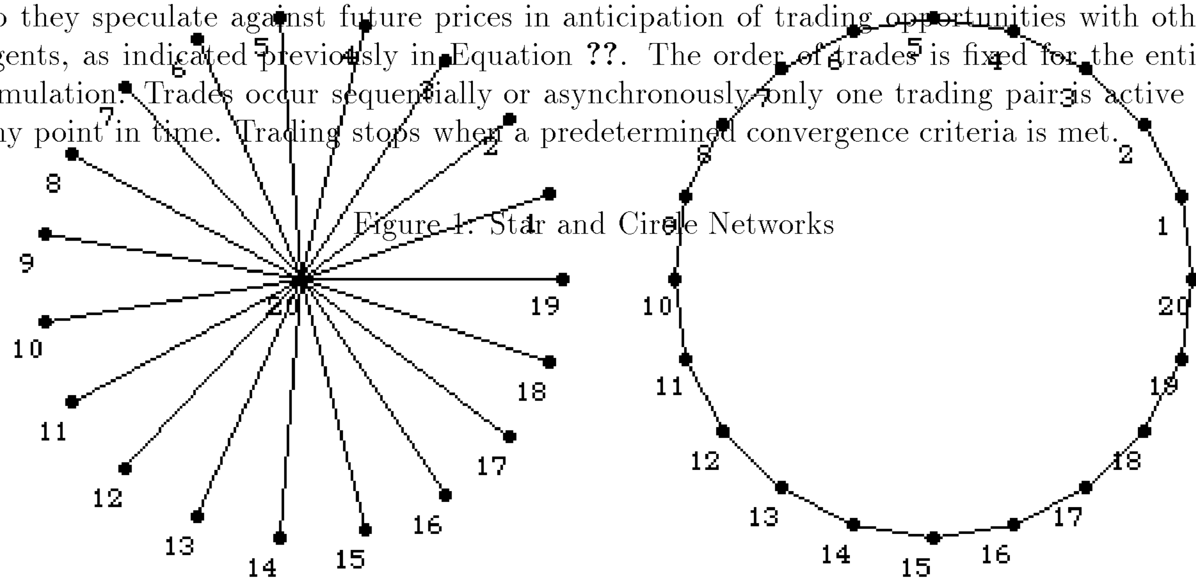
where x_{t-1}^i (y_{t-1}^i) is agent i's holdings of good x (y) before trading in period t, x_t^i (y_t^i) is agent $i's$ holdings of good $x(y)$ after trading in period t, and p_t is the price of good y in terms of good x at time t. Agents' current endowments consist of their chosen consumption bundle from their the last trade. Initial endowments (x_0^i, y_0^i) are heterogeneous across agents, although individual initial endowments sum to 100 and total endowments of the two goods economy wide are equal. The competitive general equilibrium price that corresponds to these initial conditions is always one, and initially all agents are equally wealthy with respect to that price.

The spatial relationships between agents or the network of trading possibilities are described by a graph. A graph or trading network $\mathcal{T}$ is defined by $\mathcal{T} = \{N, C\}$, where N is a set of agents (the vertices of the graph) and C is a set of bilateral trading connections (the edges of the graph). C is a subset of the unordered pairs of N. If (i, j) is an element of C then agents i and j are adjacent. A path between agents i and j is a subset of trading connections $\{(i, i+1), (i+1, i+2), \ldots, (i+m-1, i+m), (i+m, j)\}$ where $\{i+1, \ldots, i+m\}$ is a proper subset of N. A network of trades is connected if there is a path between every pair of agents.

3 Simulations

For simplicity in the initial simulations each bilateral trade between adjacent agents clears the market consisting of those two agents. The resulting allocation is simply the competitive general equilibrium solution for the two person economy. Any algorithm for selecting a point on the bilateral contract curve that gives both agents higher utility than their current endowments would serve equally well. Agents do not engage in any strategic behavior, nor do they speculate against future prices in anticipation of trading opportunities with other agents, as indicated previously in Equation ??. The order of trades is fixed for the entire simulation. Trades occur sequentially or asynchronously, only one trading pair is active at any point in time. Trading stops when a predetermined convergence criteria is met.

Figure 1: Star and Circle Networks



The dynamic behavior of the trading network is illustrated by example simulations for two simple network structures, a circle and a star. There are twenty agents, $n = 20$. Agents' preferences and initial endowments are the same in both cases, however, the connections between agents differ. In the first simulation agents are arranged in a hub–spoke or star configuration and in the second simulation in a circle (Figure ??). The number of trading connections in the star network is $n-1$, or 19, the number of trading connection in the circle network is n, or 20. Clearly, the star network is more centralized: the average distance or path length between agents is 1.9, the median and maximum are 2. In contrast, the average path length on the circle is 5.26, the median is 5, and the maximum is 10.

3.1 Star

Figure ?? shows logged prices by trade. Observed prices by trading pairs (agents' marginal rates of substitution) were within 0.05 percent of each other after 21 rounds of trading or 399 individual trades. The maximum price in the final round of trading was 1.1024 and the minimum was 1.1019. Recall that the competitive general equilibrium price that corresponds to the initial endowments and preferences is 1. As the figure indicates, prices converge roughly exponentially. Figure ?? shows the log of prices displayed by trading pair. The horizontal axis corresponds to trading pairs $(1,20),(2,20),...(19,10)$. As prices and agents' marginal rates of substitution converge, trades occur close to a horizontal line at $p = 1.1$.

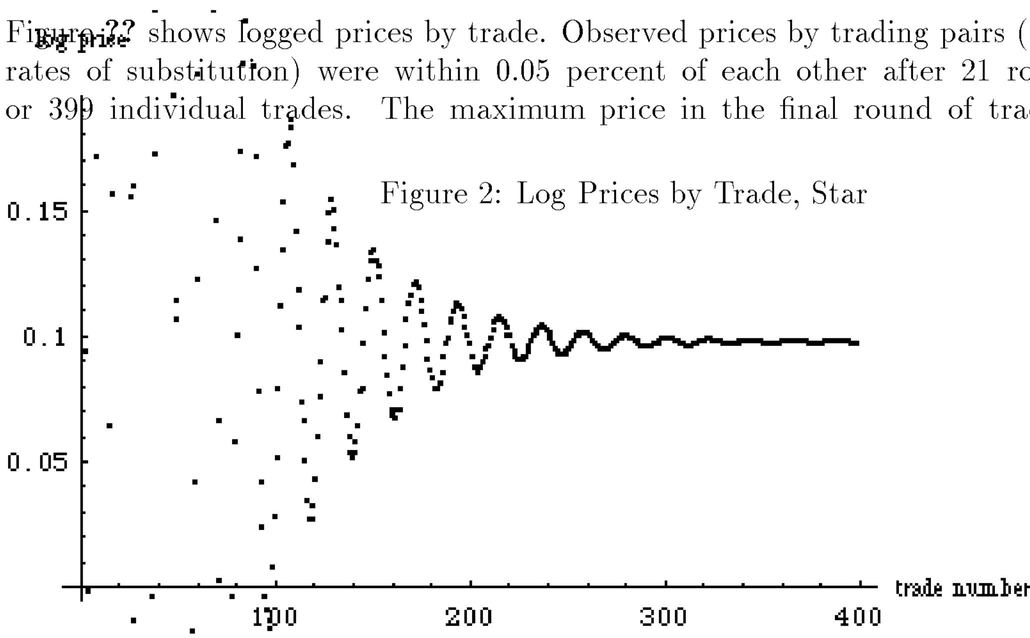


Figure 2: Log Prices by Trade, Star

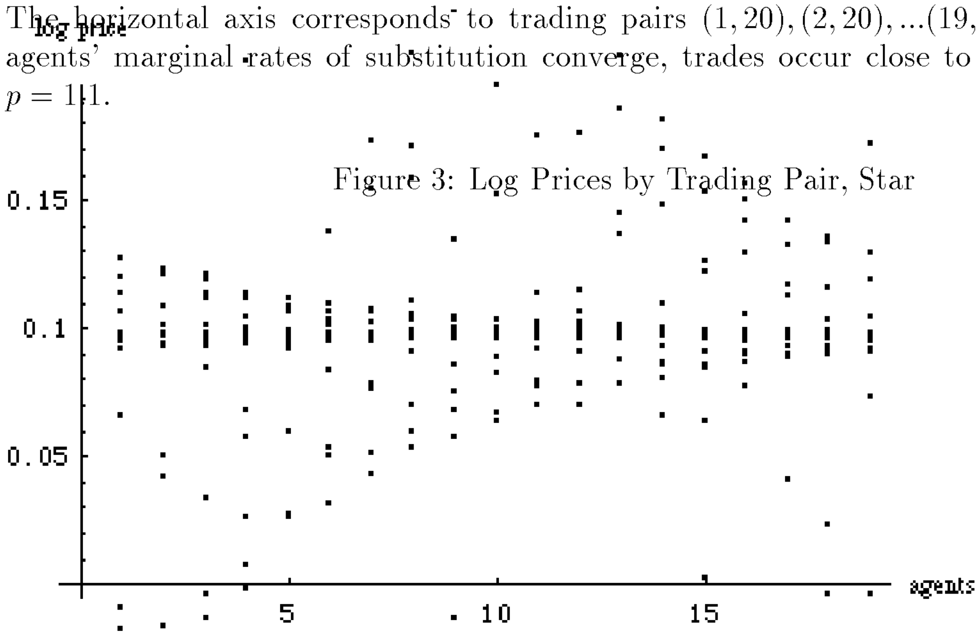


Figure 3: Log Prices by Trading Pair, Star

Figure ?? shows the coefficient of variation for all agents' marginal rates of substitution after each trade. This series is used to compare rates of convergence across simulations.

The last two thirds of the coefficient of variation data were fit to an exponential function using ordinary least squares. The magnitude of the estimated exponential parameter in this example is 0.022.

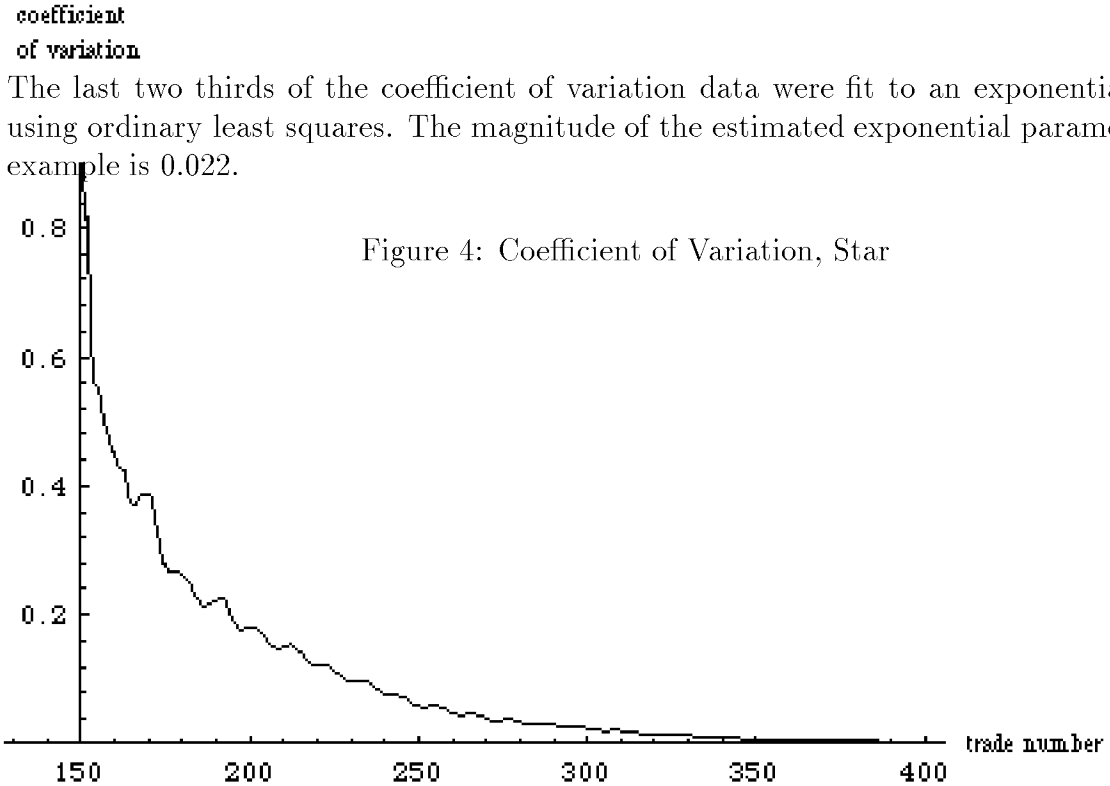


Figure 4: Coefficient of Variation, Star

3.2 Circle

Figure ?? shows logged prices by trade for the circle network. Note that agent's preferences and initial endowments are identical to those used in the previous simulation. The relative location of agents in the two networks is shown in Figure ??.

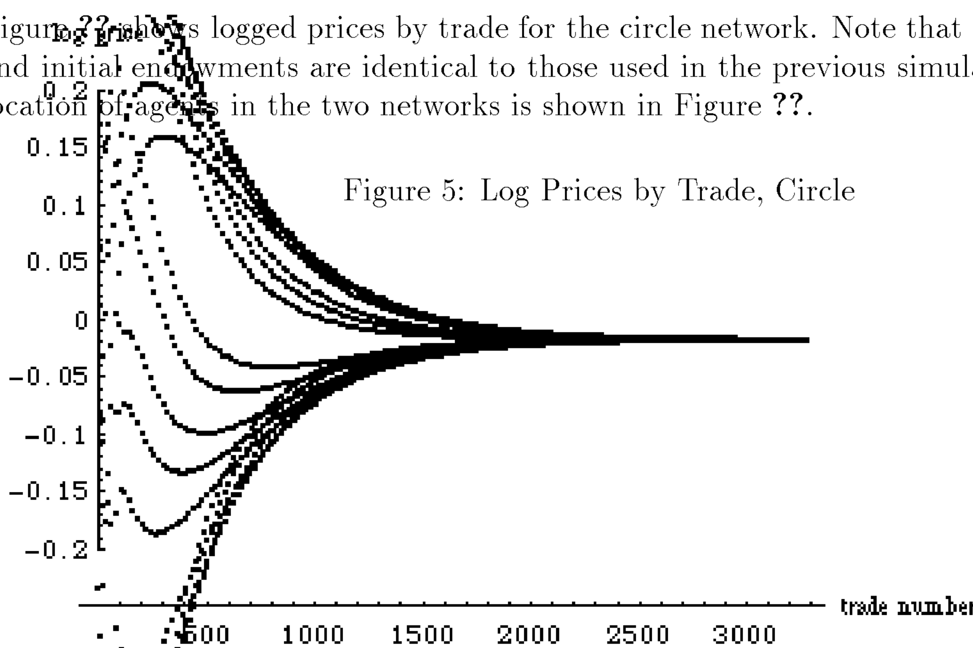
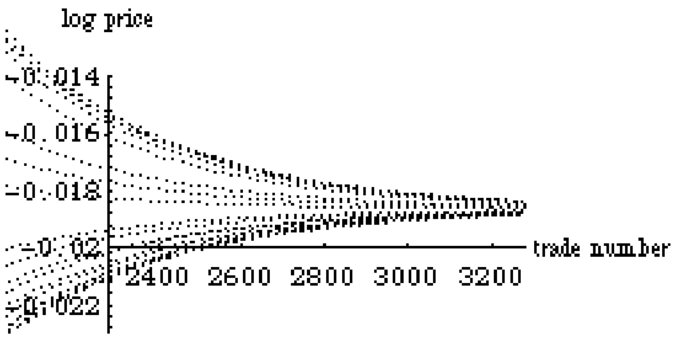


Figure 5: Log Prices by Trade, Circle

Observed prices by trading pairs (agents' marginal rates of substitution) were within 0.05 percent of each other after 164 rounds of trading or 3280 individual trades, nearly eight times longer than the star network configuration. The maximum price in the final round of trading was 0.9818 and the minimum was 0.9813. In contrast to the previous simulation, the convergence of agents' marginal rated of substitution is not smooth across space. Different regions of the lattice converge to distinct, spatially-determined price ranges which persist over time, even though the entire economy eventually achieves a common valuation of the goods. The visible lines in Figure **??** represent the price used by a particular trading pair over time, the persistent spatial variations in prices can be seen in the one large and another smaller gaps between these prices. Furthermore, these gaps persist over time and are still identifiable in the last 50 rounds of trading.

Figure 6: Log Prices for Last 1000 Trades, Circle



The spatial variation in logged prices over time can be observed in Figure **??**. The gray value of each square represents the logged price of a bilateral trade. Agent trading pairs are arrayed horizontally and trading rounds are arrayed vertically. Each succeeding row displays the trade prices of every trading pair for a complete trade round. Figure **??** emphasizes this persistence of spatial correlations in prices. (Note that the price scale for this figure runs from -0.484 to 0.439.) The vertical lines that occur between trading pairs $(1, 2)$ and $(2, 3)$ and $(12, 13)$ and $(13, 14)$ arise because agents 2 and 13 consistently trade small amounts of goods at widely different prices with their two neighbors. Consequently, these agents act as temporary barriers that slow the movement of goods across the lattice and the convergence of agents' marginal rates of substitution. The emergence of such barriers depends not only on the characteristics of the agents on the boundary but also on the characteristics of his or her nearest neighbors. For example, the formation of a boundary is more likely when an agent has preferences skewed in favor of one of the goods (an α parameter of say .8 or .2 in Equation **??**) but the quantity of the agent's holdings of goods relative to neighboring agents also plays a crucial role. In the initial rounds of trading, trades occur at prices far from the equilibrium price. These trades often redistribute large amounts of goods between agents. In subsequent trades the market clearing price and quantity traded depends critically on the relative size of the endowments of the two agents. The order of trades in the initial rounds affects the scale, size and direction of these redistributions. This demonstrates one way the process of convergence can exhibit path dependence: an initial unfavorable trade that causes one agent to be relatively less wealthy than his or her neighbors can result in persistent spatial variations in prices and slow convergence of the entire network.

Figure **??**, which shows log prices by trading pair, displays both the transient spatial correlations in price and convergence to common price over time. Figure **??** shows the coefficient of variation for all agents' marginal rates of substitution after each trade. The magnitude of the estimated exponential parameter for the circle network is 0.0024, an order of magnitude smaller than in the simulation of the star network.

3.3 Other Network Structures

The slow convergence and spatial correlations observed in the example simulations of the circle network are not unique to that network configuration. Large scale simulations of a variety of networks with 40 agents demonstrate that the speed of convergence is closely related to the degree of centralization of the network. Furthermore, slow convergence occurs in conjunctions with the long term persistence of distinct neighborhood price regimes or spatial correlation in local prices.

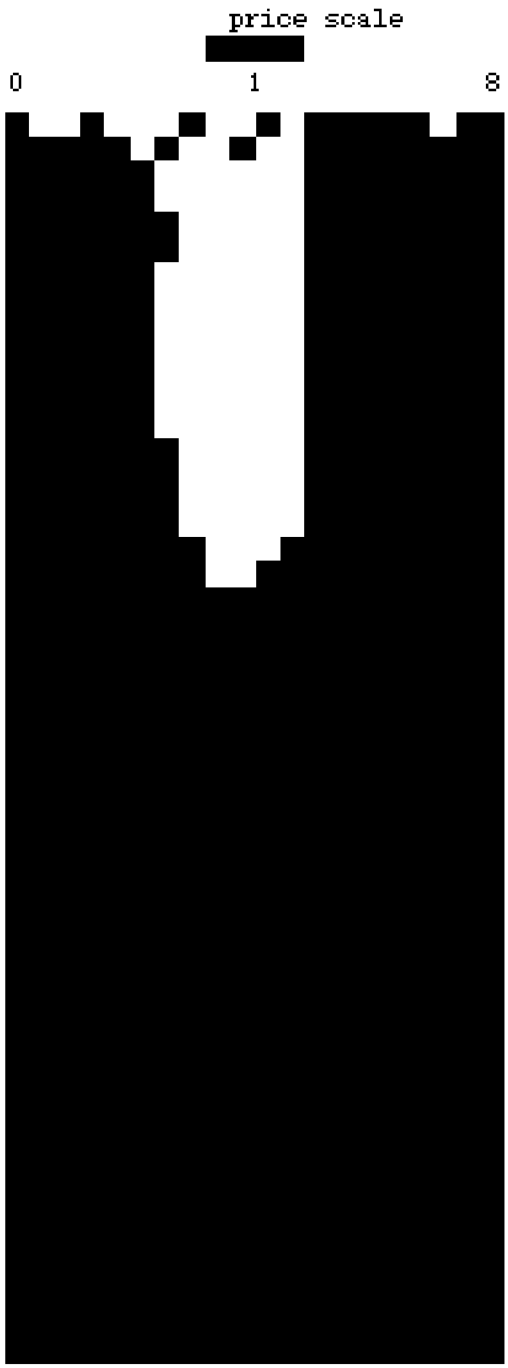
In addition to the circle and star three other types of networks were considered. The crystal type network were generated by adding agents to the network one a time with an equal probability of connecting to any agent already in the network: the third agent added to the network is connected to the first or the second agent in the network with a probability of one-half each; the fourth agent added to the network is connected to one of the first three agents with a probability of one third each; and so on. The crystal type networks are relatively centralized, with the agents added to the network early on occupying the more centralized locations. The number of connections in the crystal networks was $n-1$ or 39.

The hub type networks are also quite centralized. They were generated from an initial ring structure (a small circle of 5–8 agents) with all subsequent agents being connected to one of the agents on the hub. The number of connections for the hub structure is n or 40. The hub structure was motivated by previous theoretical work on bilateral trading networks (Bell [?]).

The networks labeled 'random' were generated by randomly removing connections from an initially complete network with connections between all pairs of agents. Connections were removed one by one, checking to make sure that the network remained connected after step. The random networks have n connections and are significantly less centralized than the other two types. Figure **??** summarizes the data (mean, median and maximum path length between all possible pairs of agents) for these network structures. Figure **??** summarizes the data on speed of convergence for approximately 350 runs with different preferences, initial endowments and order of trades for each of the various network structures.

Figure **??** relates the number of trades required for convergence to the mean path length between all possible pairs of agents for these networks. (Figures **??**, **??** and **??** appear at the end of the paper.) Each vertical line shows the maximum and minimum number of trades for one network structure. The tick mark on the vertical line shows the average number of trades for that network structure. Figure **??** relates the estimated exponential convergence parameter to the mean path length for the same simulations. The average path length is a robust predictor of the speed of both measures of convergence. Note that the

Figure 7: Spatial Distribution of Log Prices, Circle



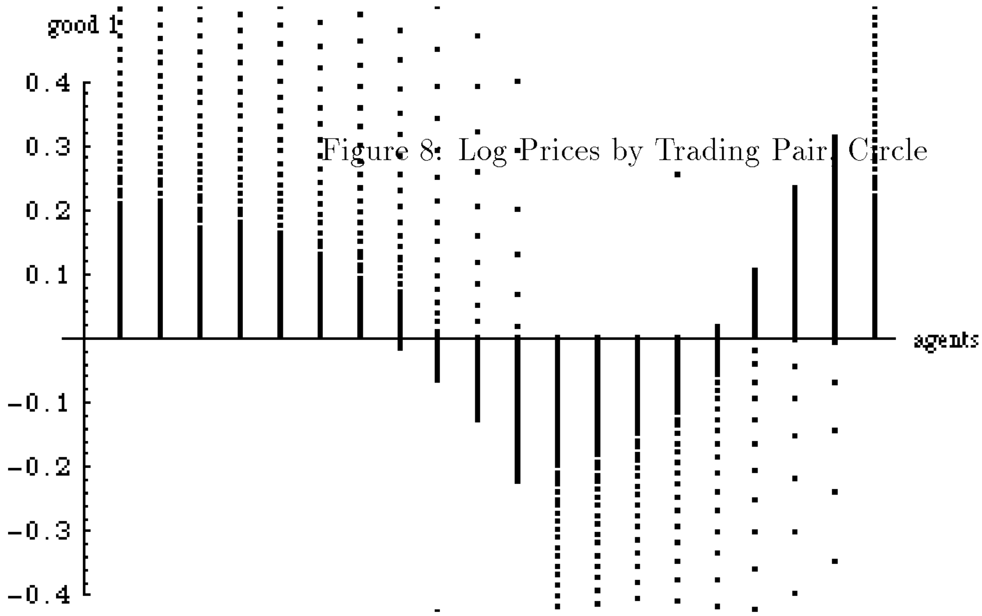


Figure 8: Log Prices by Trading Pair, Circle

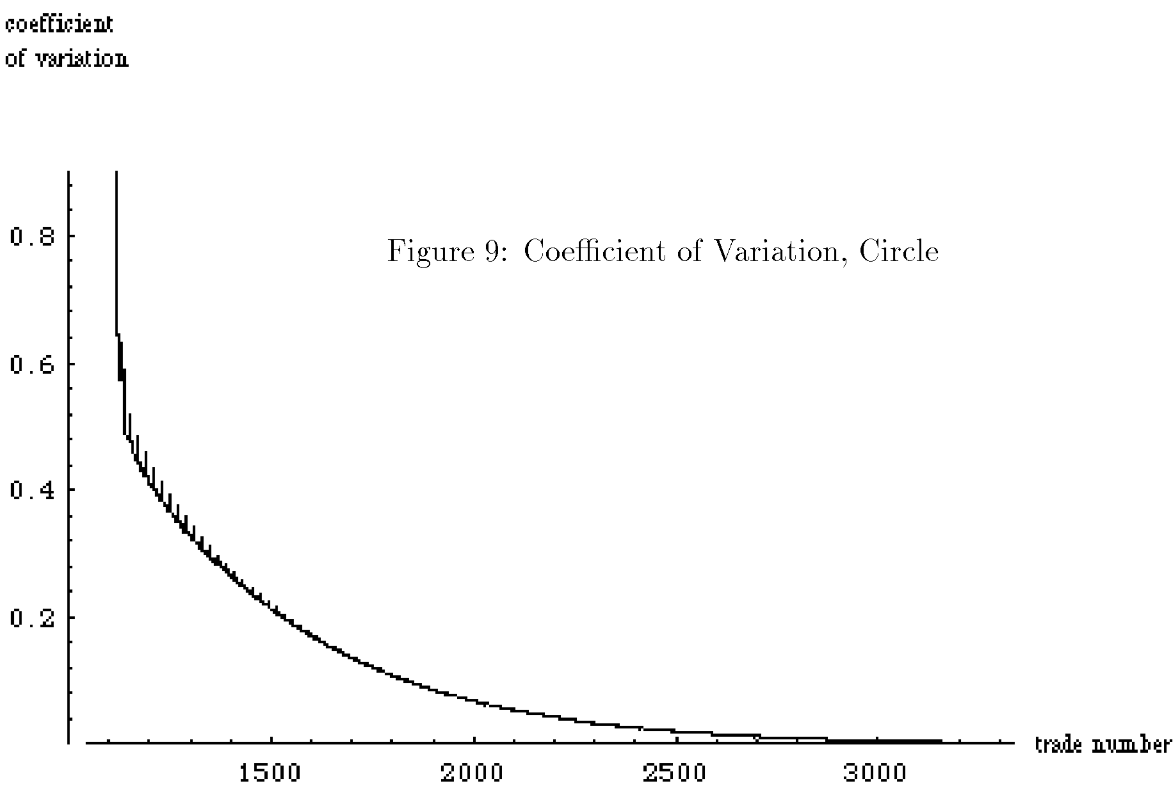


Figure 9: Coefficient of Variation, Circle

Figure 10: Network Structure

Network	**Mean**	**Med**	**Max**
star	1.95	2	2
crystal1	3.00	3	7
crystal2	3.36	3	6
hub1	3.35	3	7
crystal3	3.40	3	7
hub2	3.65	4	6
hub3	4.00	4	8
random1	4.96	5	12
random2	5.21	5	12
random3	5.48	5	12
circle	10.26	10	2

variance of the speed of convergence increases dramatically with both the path length and the speed of convergence. In some cases relatively decentralized networks exhibit fast smooth convergence to a Pareto efficient allocation. Furthermore, they do not exhibit any persistent spatial correlations in local prices. The slow convergence is associated with the persistent spatial variations in local trading prices documented for the example simulation of the circle network considered above. Figure **??** shows the log of prices for one simulation using the 'hub3' network; the distinct neighborhood price regimes are readily apparent.

4 Conclusion

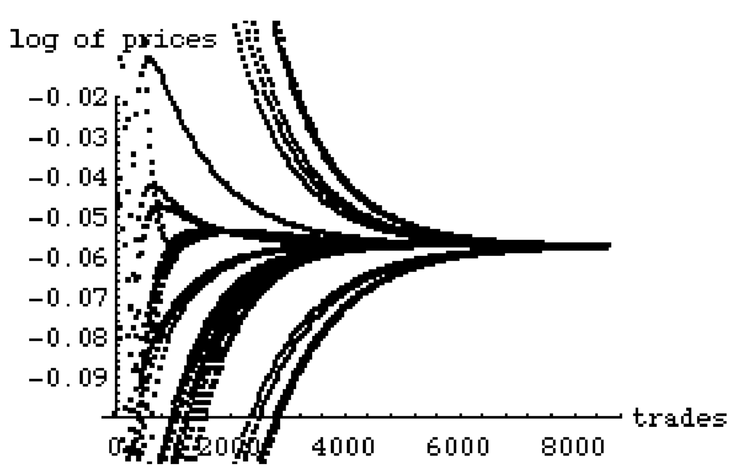
This paper examines the dynamics of a decentralized trading network through the use of computer simulations. Different trading networks exhibit widely varying speeds of convergence and interesting intertemporal and cross-sectional dynamics. The spatial structure of the network as well as the details of the trading process such as the order of trades interact in determining the speed of convergence. Not surprisingly, centralized networks converge faster and are less prone to persistent spatial anomalies in local prices.

Extensions include an examination of a broader range of bilateral trading mechanisms. Further research will relax the assumption of myopic agents by introducing boundedly rational agents who form limited time expectations of future prices and future trading opportunities. This may lead to faster convergence as trading opportunities and price differentials are exploited more quickly. However, it also introduces the possibility of local price bubbles that form when one agent with highly unrealistic expectations of future prices transmits those expectations spatially to his or her trading partners, who transmit them to their neighbors, and so on. The case of perfectly myopic agents presented here forms a good benchmark for this future research.

Figure 11: Speed of Convergence Across Networks

Network	**Mean Trade**	Max Trade	Min Trade	Mean Exp	Max Exp	Min Exp
star	1365	3783	741	−0.00716	−0.00212	−0.01316
crystal1	2063	5720	1040	−0.00531	−0.00141	−0.01261
crystal2	2216	5400	1240	−0.00449	−0.000142	−0.00865
hub1	3049	7840	1360	−0.00313	−0.00090	−0.00950
crystal3	2489	4800	1320	−0.00394	−0.00180	−0.00757
hub2	2314	3720	1320	−0.00413	−0.00217	−0.00744
hub3	7016	17600	1560	−0.00127	−0.00038	−0.00666
random1	7040	13720	3880	−0.00134	−0.00058	−0.00268
random2	7547	16160	4160	−0.00124	−0.00051	−0.00209
random3	10153	22560	3760	−0.00093	−0.00036	−0.00046
circle	17004	24360	8320	−0.00047	−0.00029	−0.00091

Figure 12: Log Prices by Trade, Hub 3



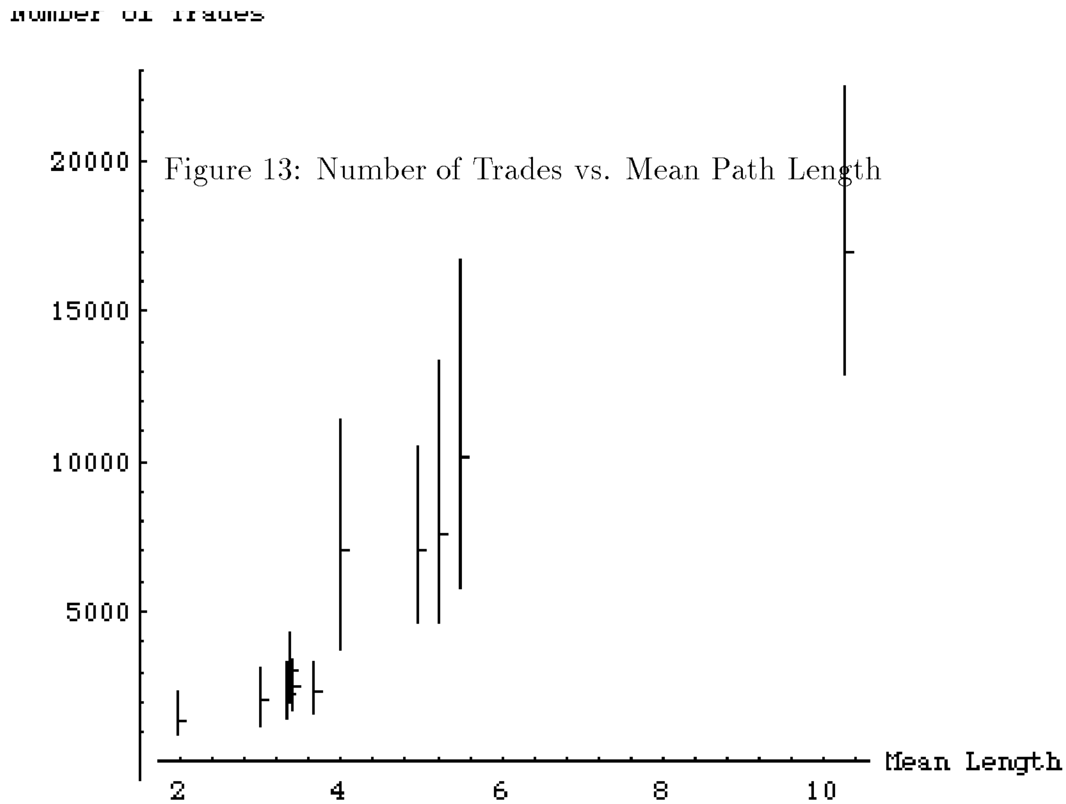


Figure 13: Number of Trades vs. Mean Path Length

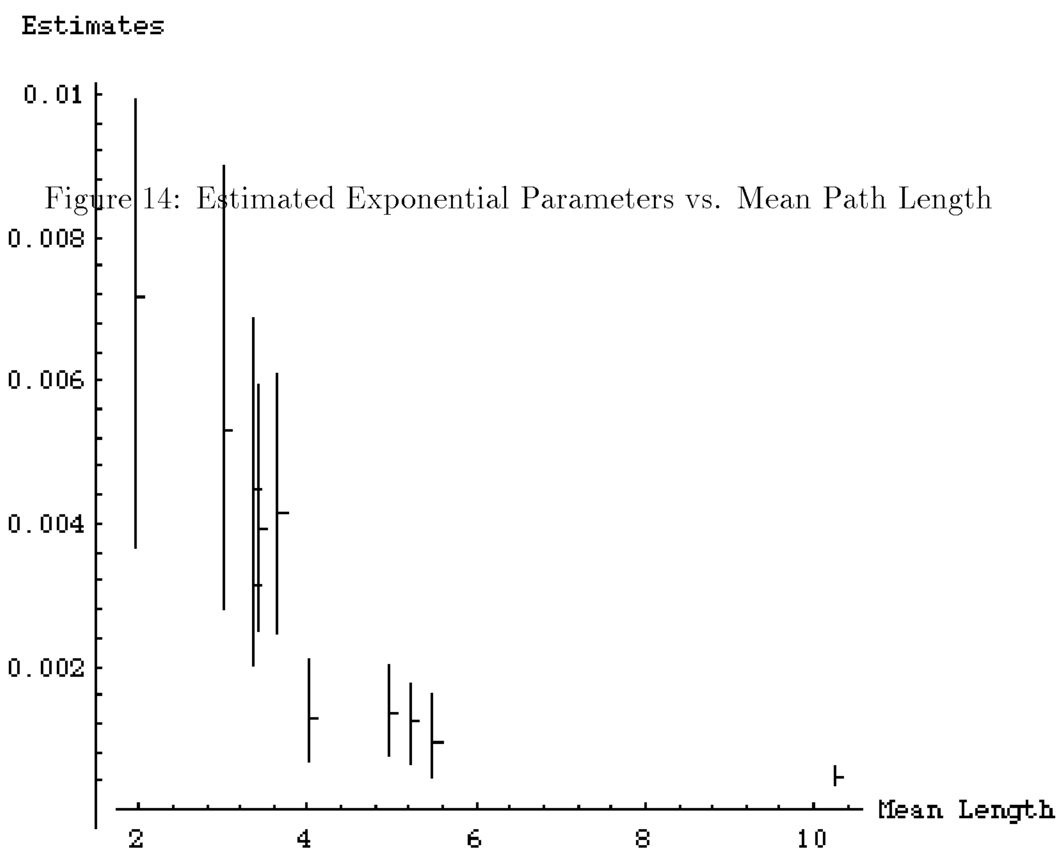


Figure 14: Estimated Exponential Parameters vs. Mean Path Length